CASE STUDY

Developing a composite-based constitutive model for municipal solid waste

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BACKGROUND AND OBJECTIVES: Harmful ruptures and instabilities in landfills in recent years have highlighted the importance of studying the municipal solid waste and its behavior. These instabilities mostly occur in the landfill of developing countries where waste materials are degradable and saturated. The behavior of waste and its ingredients are unknown as the main reasons for such instability. The main goal of this study was to better predict the behavior of landfills and unknown materials in municipal solid waste to prevent the environmental disasters.

METHODS: A cylindrical specimen was modeled and subjected to triaxial test loading conditions using the finite element method. Also, fresh waste, as a waste sample with a specific composition, was investigated. Using the optimization method, the constants of the presented equation were obtained and the basic model of stress strain was presented based on composite theory.

FINDINGS: The whole models for predicting the waste behavior were presented based on the behavior models of soils. This was carried out by the theory of composite materials, which was used for the first time in this study. At the strains of less than 30 percent, a well agreement was observed between the results of the numerical and the present methods. Also, at confining stresses less than 100 kiloPascal, the root mean square of the relative error percentages between the total stresses obtained from the present model and another model was less than 10 percent. At higher confining stresses, this amount was in the range of 10 – 20 percent.

CONCLUSION: The results of this study were compared with those of the experimental data in previous models to verify the proposed model. The model proved to be capable of simulating and predicting the municipal solid waste behavior under various loading conditions efficiently. The results implied that assuming the municipal solid waste as composite material was reasonable and could be extended to future studies.
INTRODUCTION

In recent decades, landslides in landfills have caused significant problems, including severe environmental pollution, urban infrastructure destruction, landfill destruction, and, most importantly, human casualties (Bezerra de Araujo Filho et al., 2014; Faitli et al., 2015; Malave et al., 2020). For instance, the rupture of the Payatas landfill in the Philippines killed about 300 people. The increase in waste leachates due to rainfall is the main cause of such ruptures (Merry et al., 2005). Various methods are provided for separation, collection and storage of waste (Teshome et al., 2023). While the needs of society have been to prevent these incidents and improve environmental safety, the existing knowledge regarding the behavior of such materials is very limited (Stark et al., 2009; Kumar et al., 2009; Chelliapan et al., 2020). Predicting the behavior of materials is particularly important in various sciences such as mechanics (Liu et al., 2021; Savaedi et al., 2022), civil engineering (Sadrenejad et al., 2019; Thakur et al., 2022; Sarmah et al., 2021), biological experiments (Zhang et al., 2021; Holzapfel et al., 2021; Housan et al., 2021), and other applications. Compared to soil mechanics, the mechanical behavior of landfills has been subjected to many studies for the last three decades (Carvalhod et al., 2004; Machado et al., 2002; Kolekar et al., 2016). Most of the researchers have attempted to investigate the mechanical behavior of such materials using the laboratory investigations (Gharabaghi et al., 2008; Zekkos et al., 2005; Collin et al., 2018). The desired deformations have been obtained in the laboratory studies conducted using the triaxial test and by applying stress on the waste samples (Keramati et al., 2018; Shariatmadari et al., 2017). Due to the great complexity of parameters influencing waste behavior, there are still many unknown features in this area, as waste materials show different behaviors compared to other materials (Keramati et al., 2019; Alidoust et al., 2021). Considering the necessity for more accuracy in the prediction of such materials, numerical studies have also been carried out in recent years (Karimpour-Fard et al., 2021; Raviteja et al., 2021). Eventually, studies on municipal solid waste (MSW) materials grew in number, and researchers became more familiar with the mechanical properties of waste materials using laboratory data. As a result, the need to predict waste behavior and the role of its mechanical properties received greater attention (Shariatmadari et al., 2009; Shariatmadari et al., 2014). Some studies showed that many parameters, such as environmental conditions, confining stresses, age and temperature of the waste, had a great impact on the behavior of these materials (Karimpour-Fard et al., 2011; Alidoust et al., 2018; Abu-Qdais et al., 2020). Researchers also attempted to have a better prediction of landfills by considering the influencing parameters (Carvalhod et al., 2004; Keramati et al., 2020). One of the early behavioral models presented for modeling the waste mechanical behavior is the model suggested by Machado et al. (2008) who established their model based on triaxial experiments (Machado et al., 2008). These researchers considered waste as a two-phase substance. The first phase was supposed to be its paste-like segment originating from leachates, and the second phase was considered to be fiber. In their model, they did not consider the effect of the ratio of the waste constituents on its mechanical behavior; they relied on Young’s modulus as a constant variable through time. Machado et al. (2010) attempted to upgrade their experiment by studying fresh and 4-year-old waste. In another study, they validated their model by laboratory findings and applying a correction compensating for changes in the shape of waste materials. Nevertheless, they merely compared their model with laboratory results without predicting a comprehensive behavioral model (Machado et al., 2017). However, the presented model significantly differed from laboratory results, especially in high strains. They explained that abnormal pulses were observed in some cases in high volumetric strains, which had not been presented in the related charts. The model used by these researchers was the low-total soil model (Machado et al., 2017). Babu et al. (2010) investigated the waste behavior model using a modified low-total soil model. They assessed fresh, 1.5-year-old artificial waste (Marques et al., 2003). In their model, the obtained strains were assessed according to the soil behavior models, and the total strain of waste was calculated by summing up all the strains. The model showed a good agreement with laboratory results in small deformations and low strains. In high strains, and especially when there was an extreme all-around pressure, inconsistency was observed between the model and the laboratory results. This could be attributed to ignoring the deformations caused by
waste particles. The main problem of the model was considering a soil model for the whole substance. Krase et al. (2007) also used the theory of fibers and paste to propose their behavioral model. Later, Zhang et al. (2010) investigated the waste behavior model by dividing waste into different phases. Considering a modified low-total behavioral model for the leachate phase of waste and a one-dimensional plastic model for the solid phase of waste, they presented a new model by combining the two models. The new model had several fundamental weaknesses, including one-dimensionality, lack of suitable agreement with laboratory results in significant deformations, and, as acknowledged by the researchers, inappropriateness of the finite element model used (Zhang et al., 2010). Excluding the enclosed pressures of the pore water and ignoring the elastoplastic method could also be mentioned as other limitations of the model proposed by Zhang et al. (2010). Applying the soil model for waste causes errors in the deformation of its internal particles and alterations in the enclosed volumes. Fang et al. (2016) using the laboratory results of Babu et al. (2010) investigated waste behavior by applying the theories of biological, elastic, plastic, and consolidation strains. They presented their findings employing a parameter that improved biological strains (Feng et al., 2016). Hanke et al. (2017) compared the prediction models of waste behavior and deformations occurring in the landfill mass. This comparison was not made in the same conditions, and the results obtained from landfill strains were compared as a whole. As already mentioned, various parameters, including temperature variations of waste due to various internal interactions, can affect waste behavior (Shariatmadari et al., 2014; Machado et al., 2010). Hubert et al. (2016) comprehensively studied the strain-stress properties of waste. They presented the findings of previous studies in a graph, demonstrating that the shape changes of the landfill occurring over time at different temperatures. According to their study, temperature varied considerably during the first year of landfilling and gained more stability after about five years. Magyar et al. (2015) investigated the temperature changes of waste in a landfill. They poured waste into a metal box with a volume of 1.5 cubic meters and assessed the effect of temperature changes on waste components. Kumar et al. (2020) also studied waste behavior using the theory of soft soils. They presented their model by analyzing the behavior of soft clay, as an alternative for waste, and summing up the strains caused due to elastic, elasto-plastic, and sliding events occurring over time. They used the Mohr-Coulomb method to examine the strains created in a trench over 40 years. The results showed no specific pattern for changes in waste settlement over time and an increase in waste settlement over time (Kumar et al., 2020). The theory of composite materials in soil mechanics is used to investigate the behavior of reinforced soils and soil-plastic composite materials. The study conducted by Consoli et al. (2007) is among such experiments. They used the combination of sand and fiber as a composite material. Using the direct shear test, creating tension in the fiber, and mathematically calculating fiber movement in soil samples, they obtained the stress-strain diagram. They finally found that the angle of fiber placement in soils profoundly impacted the composite material’s behavior. Chengizadeh and Nikraz (2011) also performed direct shear tests on composite soil samples to assess the effects of these materials on soil resistance. Investigating the effects of fiber on clay soils, they witnessed shape changes in the soil after altering the percentage of fiber from 0 percent (%) to 2%. In another study, Cheng et al. (2015) used a combination of sand and fiber as a composite material. They obtained the stress-strain diagram by calculating fiber orientation in the soil sample. The famous models for the prediction of the MSW behavior are shown in Table 1.

Referring to Table 1, in most of the studies, to better predict the waste behavior, MSW has been divided into two parts: paste and fiber. Behavioral models of soils are considered as the primary model, and the fiber part is added to it. These two parts have special characteristics and behaviors, but their combination may have different characteristics. Due to the complexity of waste material, it can lead to significant errors. The presented models have been compared and evaluated using the laboratory results obtained by the own researchers, and no comprehensive behavioral model for waste has been presented so far. Since MSW is composed of unknown materials and is very different from soil, the composite theory can be helpful in providing a unified model for evaluating the behavior of waste. In this study, the theory of composite materials was used to assess the behavior of MSW. The waste
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A sample was simulated as a cylindrical shape using the finite element method for better simulation and comparison of results. Subsequently, the processes of presenting and predicting the waste behavioral model and requirements for such predictions were explained. The primary stress-strain relationship was presented for these materials using the theory of composite materials. The aim of this study was to use composite theories in predicting waste behavior. Development of this theory in future studies is suggested, as it can lead to better prediction of MSW and investigation of more parameters. This study and its numerical simulation were carried out using the data of Tehran landfill in Iran in 2021.

MATERIALS AND METHODS

In this study, fresh MSW was initially modeled using ANSYS software and then subjected to triaxial loading using the material composite theory. Moreover, a macroscopic viewpoint of composite materials was used to predict the total MSW behavior via composite-based constitutive models. After obtaining stress-strain results, the constants of composite equation were determined based on optimization methods and laboratory results. The results were compared with those of other studies to validate the equation.

Stress-strain response

ANSYS is commonly used to obtain the stress-strain response for the MSW. As shown in Fig. 1, to predict the behavior of these materials, a cylindrical sample with a diameter of 20 centimeter (cm), height of 40 cm, and fixed boundary condition at one side was simulated according to the experimental specimen conditions of the triaxial tests and subjected to compressive stress. The same confining pressure was applied to the surrounding area of the sample. The pressure started from zero and continued until the confining pressure stress was reached. After the confining pressure reached the desired value, an additional stress was applied to the upper surface of the cylinder and this value continued linearly. The strain corresponding to the stress applied on the upper surface of the sample was also obtained.

After modeling, the waste sample was subjected to meshing using tetrahedral meshes. In order to render the experiments independent of the mesh size, their dimensions were subjected to sensitivity analysis (Table 2). During the sensitivity analysis, the all-around pressure was considered as 100 kPa, and the additional stress was calculated as 150 kiloPascal (kPa). The most significant mesh value was obtained as 0.025 milimeter (mm), which was an acceptable error rate (i.e., <1%), and it was used as a basis for further calculations.

As already mentioned, the used model was composite behavior considering the elastoplastic and plastic waste behaviors and the Voce’s theory (Voce 1948). In this model, which relies on the Drager-Prager hardening model, the changes in plastic strains have
a linear relationship with rebound stresses. Using an equation relating rebound stresses to plastic strains, Armstrong and Frederik convert the linear kinematic hardening model of Prager to a non-linear model. The first kinematic hardening model was presented by Prager, in which the changes in plastic strains had a linear relationship with rebound stresses using Eq. 1 (Zhang et al. 2015; Voce 1948).

\[
dX = \frac{2}{3} C d \varepsilon - \gamma X dp
\]

(1)

Where, C and gamma are the constants of the material; and \(dp\) represents the cumulative plastic strain that has a non-linear relationship with stress. This behavioral model has been used for composite materials, and the graphs obtained from this model consist of three segments: elastic, elastoplastic, and plastic. According to the results, the waste seemed to first act in an elastic manner, then entered the elastoplastic phase, and underwent hardening. The strain-stress function of waste can be calculated from the relationships in this theory using Eq. 2 (Zhang et al. 2015; Voce 1948).

\[
\sigma = a + b_1 \left(1 - \exp \left(-\frac{-\varepsilon_p}{c_1}\right)\right) + b_2 \left(1 - \exp \left(-\frac{-\varepsilon_p}{c_2}\right)\right)
\]

(2)

Where, \(a, b_1, b_2, c_1,\) and \(c_2\) are the constants of the equation; and \(\sigma\) and \(\varepsilon_p\) represent stress and strain, respectively. This composite model was used as a reference for modeling the waste behavior. This equation was coded parametrically and modified by optimization methods to be used as a behavioral model for the simulated sample. Stress was calculated at any point of strain based on the laboratory results using numerical methods. Considering the minimal calculation error, the best prediction for waste behavior was projected. After applying all-around stress and reaching balance in the sample, the linear axial loading was executed, and the stress continued up to 30% strain. Strains greater than 30% reflect extensive deformations, which were not investigated in this study. The used fresh waste and its features are shown in Table 3. The amount of fiber combined with the laboratory sample was obtained as 12.5%. The
physical properties of the waste are demonstrated in Table 4.

According to Table 4, MSW was composed of various materials. All ingredients had different properties with different behaviors. Application of the theory of composite materials led to an integrated prediction of waste considering the behavior of all the materials used.

**RESULTS AND DISCUSSION**

Using the theory of composite materials and numerical simulation of MSW, as a triaxially loaded specimen, the behavior of fresh MSW was investigated. The Voce’s theory was used as a theory of composite materials (Zhang et al., 2015; Voce 1948). Moreover, the results of the triaxial test performed by Shariatmadari et al. (2009) were utilized in the constitutive model (Shariatmadari et al., 2009; Machado et al., 2010). To achieve the MSW stress-strain equation, the experimental strain-stress results were compared with the results of the composite method, and the constant parameters of the equation were obtained through optimization. The fiber-free waste was analyzed and then fiber was added to the material. The strain results were used against the strain parameter for a better comparison. The paste material was considered as soil and loaded using the Cam Clay Model (MCC). As shown in Fig. 2, with the increase of deviatoric stress in the sample the strain occurred at higher stresses.

According to various modeling strategies and previous studies, Young’s modulus is among the factors influencing the waste behavior. By maintaining the limiting stress constant, Young’s modulus changed in the model (Fig. 3). According to the diagram, the changes in Young's modulus were more noticeable at lower strains; these changes did not considerably affect the strain-stress results.

After modeling the paste-like material of the waste, the modeled fibers were subjected to tension by applying stresses of 100, 200, and 300 kPa, showing plasticity (Fig. 4). According to the stress-strain diagram, the more stress is applied to the fiber, the closer it becomes to flowing.

Laboratory results and the composite equation were achieved using the optimization methods for obtaining the constants of the strain-stress equation related to MSW. Laboratory conditions must be similar to allow for comparison of the obtained equation with the models proposed in previous studies. This was initially fulfilled by coding the existing models using the similar inputs. The relation for the stress-strain was obtained using Eq. 3. (Babu et al., 2010). In this part, the simulation was done at a constant temperature and for fresh MSW.
Fig. 2: Stress-strain diagram for MSW without fiber at confining stresses of 100, 200, and 300 kPa

Fig. 3: Stress-strain diagram for MSW at Young's moduli of 5, 10, and 15 MPa

Fig. 4: Stress-strain diagram for fiber at confining stresses of 100, 200, and 300 kPa
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\[ q = a_1 \left( 1 - e^{b_1 \varepsilon} \right) + a_2 \left( 1 - e^{c_1 \frac{\varepsilon}{\sigma_3}} \right) + a_3 \]  

(3)

\[ a_1 = -2.79 - 0.07\sigma_3 - 0.0002\sigma_3^2 \]
\[ b_1 = 0.24 \]
\[ b_2 = -9.17\sigma_3 \]
\[ c_1 = -0.27 \]
\[ c_2 = -0.27 \]

Where, \( \varepsilon \) denotes the MSW total strain; and \( \sigma_3 \) is the confining stress. According to Eq. 3, the total stress was relative to changes in the deviator stress, and the strain was dependent on the confining stress. One of the models available for MSW is the one developed by Babu et al. (2010). This model was compared with the model proposed in this study. In their model, the sum of sudden strains, degradation, biological and consolidation strains was used to predict the waste behavior using soil models.

The model proposed by Babu et al. (2010) showed a good agreement with their laboratory results regarding small deformations and strains (Fig. 5). Inconsistencies were observed between the present model and the results reported by Babu et al. (2010) at high strains, especially at high confining stresses. The errors for this comparison were 10.1%, 8.2%, and 25.36 % at confining stresses of 50 kPa, 100 kPa, and 200 kPa. This could be attributed to ignoring the deformations caused by waste particles. The main problem of the model proposed by Babu et al. (2010) is relying on a soil model for the whole materials. Moreover, the soil model considers the impact of pore water pressure as a total and effective stress difference. Another comparison was also made between the results obtained in the present study and the findings of Feng et al. (2016) as shown in Fig. 6. In the cam-clay model, MSW has been divided into two parts: fiber and paste. This model, by combining the deformations caused by creep, biological, elastic and plastic, provides a relationship for strain stress. The difference between Feng’s model and Babu’s model is the extension of biological deformation of waste and consideration of a parameter for its deformation. As illustrated in Fig. 6, both models showed a good agreement and the existing difference was due to the errors in both methods. Obviously, as the confining stress increased, hardening occurred in the MSW and the slope of the graph increased. The root mean squares of the relative error percentages between the total stresses obtained from the presented model and Eq. 3 were 6% and 11.2% for confining stresses of 100 and 200 kPa, respectively.

The correlation between the results showed the validity of the presented formulation. One of the
reasons for these errors might be calculation errors or the influence of other effective factors in the stress-strain responses which were not taken into account. This correlation could be due to numerous interactions of waste during large deformations, having a great impact on the behavior of waste. At the beginning of the diagram up to a strain of about 2%, there was more consistency between the results indicating the behavior of MSW at low strains. The lower confining stress in the MSW led to lower errors. A reason for this adaptation could be the lower impact of stress on the chemical and physical interactions within the MSW. At the lower levels of stress, biological changes in the MSW were less, and the MSW interactions were less than the ones observed at higher stress levels. In soil mechanics, the solid matter and the pore water are assumed to be incompressible, and the soil’s volume change is only due to the volume change of the voids. For MSW, the situation seems much more complex. The volumetric strains of MSW under compression stem from not only the movement of the waste particles, but also from the compression of the compressible particles themselves. The MCC behavioral model has been used in most of the behavioral models for predicting the waste behavior. Due to the compact nature of the waste particles, application of the MCC model may cause a lot of errors. Waste particles also deform when deviator stress is applied. In the composite theory, the waste material is considered to be uniform and all the deformations are measured.

**CONCLUSION**

Predicting the behavior of MSW has been one of the most challenging topics in human science. Since there is no behavioral model to predict the behavior of this material, many researches are currently being conducted. MSW is composed of different materials, and the interaction of these materials greatly impacts their behavior. Studies on the behavioral model of waste mostly consider it as a soil material (not a single material) which has a combination of plastic materials. In soil mechanics, all the soil elements are assumed to be incompressible and their behavior is quite different from that of MSW. Using the theory of composite materials, the overall deformation of MSW was calculated. In this model, the MSW behavior was simplified and mainly characterized by confining stress, which significantly affected the behavior prediction. The rendered simulation showed the large effect of the confining stress on the waste behavior. At the beginning of the diagram up to a strain of about 2%, there was more consistency between the results indicating the MSW behavior at low strains. The lower confining stress in the MSW led to lower errors. At stresses less than 100 kPa,
the error percentage was obtained to be less than 10%. The interaction of internal waste materials was also less. In higher stresses, the hardening in the waste behavior was more visible. Since almost all the researchers have compared the results of the behavioral models with their own results, no comprehensive prediction of MSW behavior has been provided so far. In this study, after determining the stress-strain relationship for waste, the results were compared with the findings of other studies. Although, almost acceptable initial results were obtained in this study, it is still recommended to predict the behavior of this complex material considering other influential parameters in future studies using the composite theory. This model has been developed for fresh MSW but it can be used for early prediction of landfill behavior. Moreover, it was concluded that the use of finite element simulations and considering all the influencing parameters in the MSW behavior could lead to a more accurate understanding of these materials.

**AUTHOR CONTRIBUTIONS**

M.R. Yousefi, the first author, contributed to the simulation performing, data analysis, interpreting the results, and preparing the manuscript. A. Noorzad, as the corresponding author, the second author, and the first author supervisor, conceptualized the research and checked the manuscript for English language error. M.J. Mahmoodi participated in the revision of the manuscript.

**CONFLICT OF INTEREST**

The authors declare no potential conflict of interest regarding the publication of this work. In addition, the ethical issues including plagiarism, informed consent, misconduct, data fabrication and, or falsification, double publication and, or submission, and redundancy have been completely witnessed by the authors.

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**ABBREVIATIONS**

- %: Percent
- ε: Strain
- εₚ: Plastic strain
- γ: Constant of equation
- γₛ: Specific weight
- σ: Stress
- σ₃: Confining stress
- φ: Internal frictional degree
- κ: Constant of the Camclay equation
- λ: Constant of the Camclay equation
- a: Constant of equation
- b₁: Constant of equation
- b₂: Constant of equation
- C: Constant of equation
- c₁: Constant of equation
- c₂: Constant of equation
- cm: Centimeter
- dε: Change in strain
- dX: Change in kinematic back stress
- dP: Cumulative plastic strain
- E: Elastic module
- Eq.: Equation
- exp: Exponential operator
- Fig.: Figure
- kN/m³: Kilo newton per cubic meter
REFERENCES


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