Profit intensity criterion for transportation problems

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In this study criterion of maximum profit intensity for transportation problems, in contrast to the known criteria of minimum expenses or minimum time for transportation, is considered. This criterion synthesizes financial and time factors and has real economic sense. According to the purpose of this paper, the algorithm of solution of such transportation problem is constructed. It is shown that the choice is carried out among Pareto-optimal options, moreover the factor of time becomes defining for the high income from transportation, and the factor of expenses – at low ones. Not absolute but relative changes of numerator and denominator become important when the criterion represents the fraction (in this case – the profit intensity as the ratio of profit to time). Nonlinear generalization of such transportation problem is proposed and the scheme of its solution in a nonlinear case is outlined. Graphic illustrations of Pareto-optimal and optimal solutions of transportation problem by profit intensity criterion are also given.

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ABSTRACT

In this study criterion of maximum profit intensity for transportation problems, in contrast to the known criteria of minimum expenses or minimum time for transportation, is considered. This criterion synthesizes financial and time factors and has real economic sense. According to the purpose of this paper, the algorithm of solution of such transportation problem is constructed. It is shown that the choice is carried out among Pareto-optimal options, moreover the factor of time becomes defining for the high income from transportation, and the factor of expenses – at low ones. Not absolute but relative changes of numerator and denominator become important when the criterion represents the fraction (in this case – the profit intensity as the ratio of profit to time). Nonlinear generalization of such transportation problem is proposed and the scheme of its solution in a nonlinear case is outlined. Graphic illustrations of Pareto-optimal and optimal solutions of transportation problem by profit intensity criterion are also given.
INTRODUCTION


MATERIALS AND METHODS

In the classical transportation problem total expenses of freight transportation are considered as a criteria indicator using Eqs. 1 to 4 (Esenkov et al., 2015).

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} \rightarrow \min_{k_{ij}}
\]

(1)

\[
\sum_{j=1}^{n} x_{ij} = a_{i}, \; i = 1, \ldots, m
\]

(2)

\[
\sum_{i=1}^{m} x_{ij} = b_{j}, \; j = 1, \ldots, n
\]

(3)

\[
x_{ij} \geq 0, i = 1, \ldots, m, \; j = 1, \ldots, n.
\]

(4)

Where, the control parameters are:

\( X_{ij} \) – volumes of transportation from the supplier \( i \) to the consumer \( j, \; i = 1, \ldots, m, \; j = 1, \ldots, n \);

External parameters are:

\( m \) – number of suppliers,

\( n \) – number of consumers,

\( c_{ij} \) – unit expenses of transportation from the supplier \( i \) to the consumer \( j, \; i = 1, \ldots, m, \; j = 1, \ldots, n \)

\( a_{i} \) – freight stocks of the supplier \( i, \; i = 1, \ldots, m, \)

\( b_{j} \) – freight needs of the consumer \( j, \; j = 1, \ldots, n \).

The condition of resolvability of the problem (1) - (4) is the balance between total suppliers’ freight stocks and consumers’ needs Eq. 5.

\[
\sum_{i=1}^{m} a_{i} = \sum_{j=1}^{n} b_{j}.
\]

(5)

Beyond such traditional statement (1) - (4), there is an economic justification of a complex of transportation: expediency (profitability) of the freight transportation from suppliers to consumers is not discussed, there is only one possibility to choose its optimum way in the problem. It is adequate, first of all, to conditions of the administrative economic system, when the necessity of freight transportation is set in a directive way, for reasons of the highest order, and it is out of the model. If it interprets the problem (1) – (4) concerning the market economy, the conclusion comes that some income \( D \) from the implementation of the whole transportation complex is implicitly supposed, then at minimum expenses \( \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot y_{ij} \) it will get the maximum profit \( f = D - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot y_{ij} \).
Moreover, the optimum plan \( \{ x_{ij}^* \} \) does not depend on the value \( D \), i.e. problems of profit maximization and expenses minimization are equivalent.

Besides the above-mentioned expense lowering approach to transport optimization, the transportation problem by time criterion is also known (Esenkov et al., 2015; Hirshleifer, 1970). Under the constraints (2)–(4) the total duration of the transportation complex, which is defined by the duration of the longest carriage from among included in the plan, is minimized using Eq. 6.

\[
\max t_{ij} \rightarrow \min x_{ij}, \quad \text{subject to} \quad x_{ij} > 0 \quad \forall i, j.
\]  

Where, external parameters \( t_{ij} \) set the time, required for the transportation of freight from the supplier \( i \) to the consumer \( j \), \( i = 1, \ldots, m, \quad j = 1, \ldots, n \).

However, each of the considered approaches suffers from certain one-sidedness: either only total financial indicators are considered, despite the timeframe within which the indicators were achieved; or the duration of the transportation complex is reduced at any cost. Simple convolution of criteria (1) and (6) with previously mentioned indicators of specific weight would have formalistic character; the created indicator would not have economic interpretation. The purpose of this investigation is to synthesize financial and time factors in profit intensity criterion for transportation problem and to propose an algorithm for solving such transportation problem.

**RESULTS AND DISCUSSION**

**Profit intensity criterion**

The suggested combined criterion of profit intensity profit (that is not profit as a whole, totally from transportation complex, it is per unit of time; it is an analog of cash flow in the investment design (Sharp, 1995) or time-charter equivalent in the navigation (Adland and Alizadeh, 2018) is a successful combination of financial and time factors using Eq. 7:

\[
D = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} \\
\max_{x_{ij} > 0} \rightarrow \max_{\{ x_{ij} \}}.
\]  

The algorithm of the solution of the transportation problem by the criterion of profit intensity (7), (2) – (4) can be as follows:

2.1. The duration matrix elements are sorted \( \{ x_{ij} \} \) in decreasing order.

2.2. The next (starting with the maximum) value \( t_1 \) is chosen from this ordered set of durations. The difference with the previous value \( t_{k-1} - t_k \) defines the choice of the current size of a step of the algorithm.

2.3. The next portion of cells is blocked in the matrix of unit expenses \( \{ c_{ij} \} \). If \( t_j > t_k \), it is necessary to appropriate \( t_j \) big value \( M \).

2.4. The problem (1) – (4) is solved using a usual method of potentials with the such transformed matrix \( \{ c_{ij} \} \).

If any of cells of the optimum plan, received at the previous value \( t_{k-1} \), was not blocked at the current value \( t_k \), the optimum plan does not change, otherwise it must be recalculated (it worsens as the solution area becomes narrower). The received plan is fixed in a set of Pareto-optimal solutions under the given \( t_k \) (and the previous plan is output from it if it has equal expenses with flowing). The module is moved on to 2.2. If after the next blocking of cells the problem (1) – (4) becomes incompatible, the previous value \( t_{k-1} \) and optimum plan, corresponding to it, will define the solution of the problem (6), (2) – (4) by the criterion of time. The process 2.2 – 2.4 is completed, then pass to the module 2.5.

2.5. The choice from the obtained set of Pareto-optimal solutions the best by criterion (7) by simple comparison. Results of implementation of preparatory modules 2.2 – 2.4 of the algorithm are illustrated in Fig. 1.

First, \( t_{\text{max}} \) is selected and solve the problem (1) – (4) without blocked cells. Solution 1 is received, i.e. the solution of the transport problem by the criterion of expenses. By two rays, emerging from this point 1 parallel to the coordinate axes, the set of majorized solutions is defined (which cannot be Pareto-optimal any more). At the choice of the following (shorter) \( \ell \) the minimum (the provided blocked cells are with values \( l_j = t_{\text{max}} \)) sum of expenses increases (the solution 2, which is so far considered as Pareto-optimal too). However, with obtaining (after the next reduction of \( \ell \)) the solution 3 with the same value of total expenses (which were not affected by the time reduction) the solution 2 should be withdrawn from the set of Pareto-optimal. Also intermediate are solutions 4 and 5, and the set of Pareto-optimal (such which are not majorized by any others) is finally formed by solutions 1, 3, 6, 7, 8. Moreover the solution 8 is the solution of the transportation problem (6), (2)
Criterion for transportation

(4) by the criterion of time, as at further reduction of time and blocking of the corresponding portion of cells the problem (1) – (4) becomes incompatible. Lower bypass of majorized solutions set can be considered as the graph of the function \( C(t) \), which characterizes the dependence of minimum total expenses on time. Finding in module 2.5 of the algorithm the graph \( \max(D - C(t) / t) \) at various values of \( D \) is shown in Fig. 2. In module 2.5 – unlike all previous modules – the dependence on the size of income \( D \), that influences only the final choice of the optimum plan by criterion (7) from among Pareto-optimum, occurs. With the increase in value \( D \) the graph of the profit function \( F(t) = D - C(t) \) increases respectively. The value of the criterion of profit intensity \( F(t)/t \) in each point is defined by a tangent of an angle of inclination of the straight line, which connects the origin of coordinates with this point. It is evident, that for any value of \( D \) optimal point by the criterion of profit intensity will be such point of the corresponding graph, for which this tangent (and the angle of the straight line inclination) will be the largest among all the points of this graph. Such optimal point will be the last common point of the straight line, which the counterclockwise round will be turned the origin of coordinates, and the graph of the corresponding function of profit. (A similar approach with a rotation of the straight line round the origin of coordinates to the last common point with a polygon of conditions area is also applied in the graphic method of solution of the problem of linear fractional programming (Stancu-Minasian, 1997; Valipour, et al., 2014).

At the low-income value \( D_1 \) the factor of expenses has crucial importance as at reduction of time expenses start exceeding this income, and instead of profit it will get losses – both total, and for a unit of time. Therefore, at low values of \( D \) the transportation problem by the criterion of profit intensity becomes equivalent to the usual transportation problem on a minimum of expenses (1) – (4), the factor of time cannot be taken into account. The optimal point will be the point \( A \), which corresponds to minimum expenses (and maximum time \( t_{max} \)). At a sufficiently high value of the income \( D_3 \) – in comparison with possible range of change of the minimum total expenses – the factor of time gets crucial importance, as the profit is always positive and does not change significantly. Therefore, at high values of \( D \) the transportation problem by the criterion of profit intensity becomes an equivalent to the transportation problem by the criterion of time (6), (2) – (4), expenses lose their influence on the final choice. The optimal point will be the point \( C \), which corresponds to the minimum time \( t_{min} \).

The optimum combination of factors of expenses and time, which is reached in a certain intermediate point \( B \), is required at an intermediate value of the income \( D_2 \), \( D_1 < D_2 < D_3 \). It is noticeable, that not absolute, but relative changes of numerator and denominator (i.e. not “on how much”, but “in how many times” the profit or time increases or decreases) become important, when the criterion represents the

\[ F(t) = D - C(t) \]

Fig. 1: Pareto-optimal solutions (1, 3, 6, 7, 8) of the transportation problems by criteria of expenses and time

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fraction (in this case – the profit intensity as the ratio of profit mass to time).

The general tendency of reduction of optimum time of transportation (abscissas of points $A$, $B$, $C$) within creasing of values of the income $D_1, D_2, D_3$ can be rather accurately traced in Fig. 2. The corresponding value of total transportation expenses increases gradually. But, despite this, the value of the criterion of profit intensity grows monotonically by income $D$. However, such conclusion cannot be made about monotonous increase concerning separately taken numerator (in this case it is profit). It takes place only partially, within this transportation optimum time (and it is final, when this time already becomes minimum possible). Upon transition to another optimum time – for example, from the point $A$ to the point $B$ – with the increase of the income the size of profit can be reduced due to the advancing growth of total expenses. It is interesting to compare optimal plans by different criteria in conditions, when expected income $D$ does not exceed even minimum possible total expenses on transportation, i.e. the whole graph of profit function $F(t)$ is below abscissa axis. From the point of view of the time criterion, this circumstance does not matter and has no effect. By the criterion of profit maximization (or expenses minimization) carriages will be carried out – let with losses, but minimum possible. And only by the criterion of the maximum of profit intensity it will be favorable to extend time, connected with losses of carriages (to increase a criterion denominator at negative numerator) indefinitely, i.e. not to carry out them at all.

**Multiple version**

It is also interesting to notice a possibility of multiple statement of the transportation problem by criterion of profit intensity, when not only one value $c_{ij}$, $t_{ij}$, but a few couples of values: $(c_{ij}(s_{ij}), t_{ij}(s_{ij}))$, $s_{ij} \in S_{ij}$, $s_{ij} \in S_{ij}$, where $S_{ij}$ is the set of options for a route $(i, j)$, corresponds to each cell $(i, j)$, i.e. transportation from the supplier to the consumer can be executed, for example, with lower expenses, but for longer time, and vice versa – for a shorter time with higher expenses and so forth.

Then the criterion of profit intensity (7) turns in Eq. 8.

$$D - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}(s_{ij}) \cdot x_{ij} \max_{x_{ij} \geq 0} t_{ij}(s_{ij}) \rightarrow \max \{c_{ij}k_{ij}\}$$

To constraints of the model (2) – (4) Eq. 9 are added

$$s_{ij} \in S_{ij}, i = 1, ..., m, j = 1, ..., n$$

For the solution of the problem in multiple statements (8), (2) – (4), (9) modules 2.1 and 2.3 of the above algorithm must be appropriately transformed: 2.1'. The decreasing order is sorted duration $\{t_{ij}(s_{ij})\}$, $s_{ij} \in S_{ij}, i = 1, ..., m, j = 1, ..., n$.

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![Fig. 2: Optimum solutions (A, B, C) of the transportation problem by the criterion of profit intensity, $D_1 < D_2 < D_3$.](image-url)
2.3’. For each cell \((i, j)\), \(i = 1, \ldots, m\), \(j = 1, \ldots, n\), it is assigned \(c_{ij} = \min_{t_{ij}(s_{ij}) \leq t_k} c_{ij}(s_{ij})\).

If \(\forall s_{ij} \in S_{ij} : t_{ij}(s_{ij}) > t_k\), then \(c_{ij} = M\).

Modules 2.2, 2.4, 2.5 of the algorithm remain without changes. Such converted algorithm allows to form for every \(l_i\) the most favorable option of basic data, and then to obtain a set of Pareto-optimal solutions and to choose from it the best plan by the criterion of profit intensity.

Nonlinear generalization

Nonlinear generalization of the problem (7), (2) – (4) provides introduction of dependencies of durations \(t_{ij}\) and unit expenses \(c_{ij}\) for each route \((i, j)\) not only from transportation options \(s_{ij} \in S_{ij}\), but also from the transportation volume \(x_{ij}\). Fundamental type of dependences \(t(x)\) and \(c(x)\) at various options of transportation (1 – cheap but slow, 2 – average, 3 – expensive but fast) is shown in Figs. 3 and 4. According to Fig. 3, \(t(0) = 0\), at \(x > 0\) initially there is a leap of duration, caused by the fact of transportation, then duration increases almost linearly (stepwise, but in a limit case – it is smoothed) with increase of freight volumes, but gradually it grows faster (other transportation opportunities do not exist), i.e. functions \(t(x)\) are convex down. It is interesting that the same duration is provided when the large volume of freight is transported by fast (but expensive) option or when small volume is transported by

![Fig. 3: Options of the dependence of transportation durations on freight volumes](image)

![Fig. 4: Options of the dependence of unit expenses on transportation volumes](image)
slow (but cheap) option. Dependencies of unit expenses on transportation volumes (Fig. 4) are more complicated: \( c(0) = 0 \), at \( x > 0 \) initially there is a leap of expenses due to a constant component, then unit expenses gradually decrease due to specialization and distribution of constant expenses on the increasing volume of freight, reach a certain minimum and start growing because of necessity of use of worse resources and (or) investment for ensuring further growth of transportation volumes. The criterion of profit intensity in the non-linear case takes form Eq. 10.

\[
D - \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}(x_{ij}, s_j) \cdot x_{ij}}{\max_{i,j} (x_{ij}, s_j)} \rightarrow \max_{\{x_{ij}, s_j\}}
\]

For the solution of the problem (10), (2) – (4), (9) it is possible to use monotonous increase of \( t_{ij}(x_{ij}) \) by \( x_{ij} \) under the given \( S_i \). Therefore, by setting the next limit duration \( t_j \) with \( t_{ij}(x_{ij}) \leq t_j \), it will get \( x_{ij} \leq x_{ij<k} \), and it will obtain the transportation problem with nonlinear efficiency function Eq. 11.

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}(x_{ij}) \cdot x_{ij} \rightarrow \min_{\{x_{ij}\}}
\]

under constraints (2), (3) and bilateral constraints Eq. 12.

\[
0 \leq x_{ij} \leq x_{ij<k}, i = 1,...,m, j = 1,...,n.
\]

From the solution of the sequence of such problems (11), (2), (3), (12) at various \( t_j \) and combinations of options with \( \{s_j\} \) the optimum plan by the criterion of intensity profits can be selected. In further investigations it is planned to use intensity profit criterion, which synthesizes financial and time factors and has real economic sense, for various scientific and applied problems, as a choice of optimal seaport for transportation, determination of optimal terms of equipment replacement and so on.

**CONCLUSIONS**

Thus, the criterion of maximum profit intensity for transportation problems, in contrast to the known criteria of minimum expenses or minimum time for transportation, is considered. This criterion synthesizes financial and time factors and has real economic sense. The algorithm of the solution of such problem is constructed. It is shown that the choice is carried out among Pareto-optimal options, moreover the factor of time becomes defining for the high income from transportation, and the factor of expenses – at low ones. Not absolute but relative changes of numerator and denominator become important when the criterion represents the fraction (in this case – the profit intensity as the ratio of profit to time). Nonlinear generalization of such transportation problem is proposed and the scheme of its solution in a nonlinear case is outlined. Graphic illustrations of Pareto-optimal and optimal solutions of transportation problem by profit intensity criterion are also given.

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**CONFLICT OF INTEREST**

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by the authors.

**ABBREVIATIONS**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>a</td>
<td>freight stocks of supplier</td>
</tr>
<tr>
<td>b</td>
<td>freight needs of consumer</td>
</tr>
<tr>
<td>c</td>
<td>unit expenses of transportation</td>
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<tr>
<td>D</td>
<td>income from whole transportation complex</td>
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<tr>
<td>Eq.</td>
<td>equation</td>
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<tr>
<td>Fig.</td>
<td>figure</td>
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<tr>
<td>i</td>
<td>index of supplier</td>
</tr>
<tr>
<td>j</td>
<td>index of consumer</td>
</tr>
<tr>
<td>m</td>
<td>number of suppliers</td>
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<tr>
<td>M</td>
<td>very big value</td>
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<tr>
<td>max</td>
<td>maximum</td>
</tr>
<tr>
<td>min</td>
<td>minimum</td>
</tr>
<tr>
<td>n</td>
<td>number of consumers</td>
</tr>
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s transportation variant
S set of transportation variants
t duration of transportation
t_k limit duration of transportation
x volume of transportation

REFERENCES


